Zadatak 1 :

Izračunati vrijednost odredjenog integrala :

$$\mathbf{I} = \int_0^1 \left[\frac{4}{1+x^2} - \frac{3\cdot\sqrt{x}}{2}\right] dx .$$

$$= \int_0^1 \left[\frac{4\cdot 1}{1+x^2} - \frac{3}{2}\cdot\sqrt{x}\right] dx = 4\cdot\int_0^1 \frac{1}{1+x^2}\, dx - \frac{3}{2}\cdot \int_0^1 \sqrt{x}\, dx =$$

$$= 4\cdot I_1 - \frac{3}{2}\cdot I_2 \quad \text{....................(R)}$$

Sada računamo integrale I_1 i I_2 :

$$I_1 = \int_0^1 \frac{1}{1+x^2}\, dx = [arctg x]_0^1 = \text{arctg } 1 - \text{arctg } 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

$$I_2 = \int_0^1 \sqrt{x}\, dx = \int_0^1 \sqrt{x^1}\, dx = \int_0^1 x^{1/2}\, dx = \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^1 =$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{2}{3}\cdot \left[\sqrt[2]{x^3}\right]_0^1 = \frac{2}{3}\cdot \left[\sqrt{x^3}\right]_0^1 = \frac{2}{3}\cdot (\sqrt{1^3} - \sqrt{0^3}) =$$

$$= \frac{2}{3}\cdot (\sqrt{1} - \sqrt{0}) = \frac{2}{3}(1-0) = \frac{2}{3}\cdot 1 = \frac{2}{3} .$$

$$I_1 = \frac{\pi}{4} \quad , \quad I_2 = \frac{2}{3} .$$

Zamjenom ovih vrijednosti u gornju relaciju (R), imamo da je polazni integral I jednak :

$\stackrel{(R)}{\Longrightarrow}$:

I = 4· $I_1 - \frac{3}{2} \cdot I_2$ **=4 .** $\frac{\pi}{4} - \frac{3}{2} \cdot \frac{2}{3} = \pi - 1$

Dakle , dati (polazni) integral I = $\pi - 1$ **.**
Rješavajući integral I gore (na početku-red iznad relacije (R)) , prvo smo koristili osobinu 4., a zatim osobinu 3. odredjenih integrala (vidjeti lekciju : Osobine odredjenih integrala) .

Zadaci za vježbanje : Izračunati sljedeće integrale

2. $I = \int_0^{\frac{\pi}{2}} (\cos x + \sin x)\, dx$ **.**

3. I= $\int_0^1 (\sqrt[3]{x} - \frac{5}{2})\, dx$ **(** $\sqrt[3]{x} = \sqrt[3]{x^1} = x^{\frac{1}{3}} = x^{\alpha}$ **,** $\alpha = \frac{1}{3}$ **)**

4. I= $\int_{-2}^{+2} (x^4 - 5 \cdot x^2)\, dx$. (+2 = 2)