

Primjeri:

Koristeći osobine određenih integrala ,izračunati određene integrale :

a) $\int_0^{1/2} \frac{6dx}{\sqrt{1-x^2}}$. Ovdje ćemo koristiti osobinu 2. određenog integrala :

$$\int_a^b k \cdot f(x) dx = k \cdot \int_a^b f(x) dx; \mathbf{k=6, f(x) = \frac{1}{\sqrt{1-x^2}}}.$$

$$\int_0^{1/2} \frac{6dx}{\sqrt{1-x^2}} = \int_0^{1/2} \frac{6 \cdot 1 \cdot dx}{\sqrt{1-x^2}} = \int_0^{1/2} \mathbf{6} \cdot \frac{\mathbf{1}}{\sqrt{1-x^2}} \cdot dx = \mathbf{6} \cdot \int_0^{1/2} \frac{\mathbf{1}}{\sqrt{1-x^2}} \cdot dx = \mathbf{6 \cdot I}$$

.....(1) , gdje je $I = \int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx$.

Sada računamo **integral I** (po Njtn-lajbnicovoj formuli :

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a) ,$$

$$\mathbf{I} = \int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx = [\mathbf{arcsin x}]_{x=0}^{x=1/2} = \mathbf{arcsin \frac{1}{2}} - \mathbf{arcsin 0} = \frac{\pi}{6} - 0 = \frac{\pi}{6} . ($$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \mathbf{arcsin x} + C, \mathbf{F(x)=arcsin x}). (F(b)=F(1/2) = \mathbf{arcsin \frac{1}{2}} = \frac{\pi}{6} ,$$

$$\mathbf{F(a)=F(0) = arcsin 0 = 0 , F(b)-F(a) = \frac{\pi}{6} - 0 = \frac{\pi}{6}).$$

Iz (1) imamo da je polazni-dati integral: $\int_0^{1/2} \frac{6dx}{\sqrt{1-x^2}} = \mathbf{6 \cdot I} = \mathbf{6 \cdot \frac{\pi}{6}} = \mathbf{\pi} .$